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Fundamentals of Adaptive Noise Canceling

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13. ABSTRACT *(Maximum 200 words)*
The theory underlying a possible solution, via adaptive noise canceling, to the cosite interference problem encountered by co-located frequency hopping radios is presented. It is also shown how and why adaptive noise canceling can be used, via an adaptive line enhancer (ALE), to separate narrow band deterministic and wide band random signals. Analysis of both an adaptive noise canceler with a single input and an adaptive line enhancer are described in terms of the Wiener or optimal weights of a surface acoustic wave (SAW) programmable transversal filter (PTF) contained within these circuits. In an effort to explain how an adaptive noise canceler with a single input and an ALE actually work, the functional relationship between the optimal PTF weight values (and hence the PTF frequency response) and the interfering and intended signals is developed in much more detail than is found in textbooks or review articles. Three different adaptive algorithms (Least Mean Square, Differential Steepest Descent, and Linear Random Search) for use with these adaptive filters are also described. A SAW device implementation of a PTF that could be used in building an adaptive noise canceler with single input or an ALE is described. Performance levels (maximum input power, interference suppression, and switching speed) are given to illustrate its capabilities.

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INTRODUCTION

This report presents the theory underlying a possible solution, via adaptive noise canceling, to a cosite interference problem encountered by co-located frequency hopping radios. When two or more such radios and their antennas are independently operated in close proximity, i.e., in a jeep or communication shelter, a cosite interference problem can develop. In this type of situation, the radio may not be able to meet its specified bit-error-rate. A degraded bit error rate means that the radio receiver's sensitivity will be degraded, which results in a decreased communications range.

This type of interference problem is caused by the transmitter's strong signal being too close to the frequency of the desired, weaker signal, trying to be received. The difference in power levels between the strong interfering transmitter signal at the receiver input and the minimum signal the receiver is capable of detecting could be in excess of 130 dB. For more details on a typical cosite scenario (signal and interfering power levels, frequency separation, required suppression, etc.) see Reference 18.

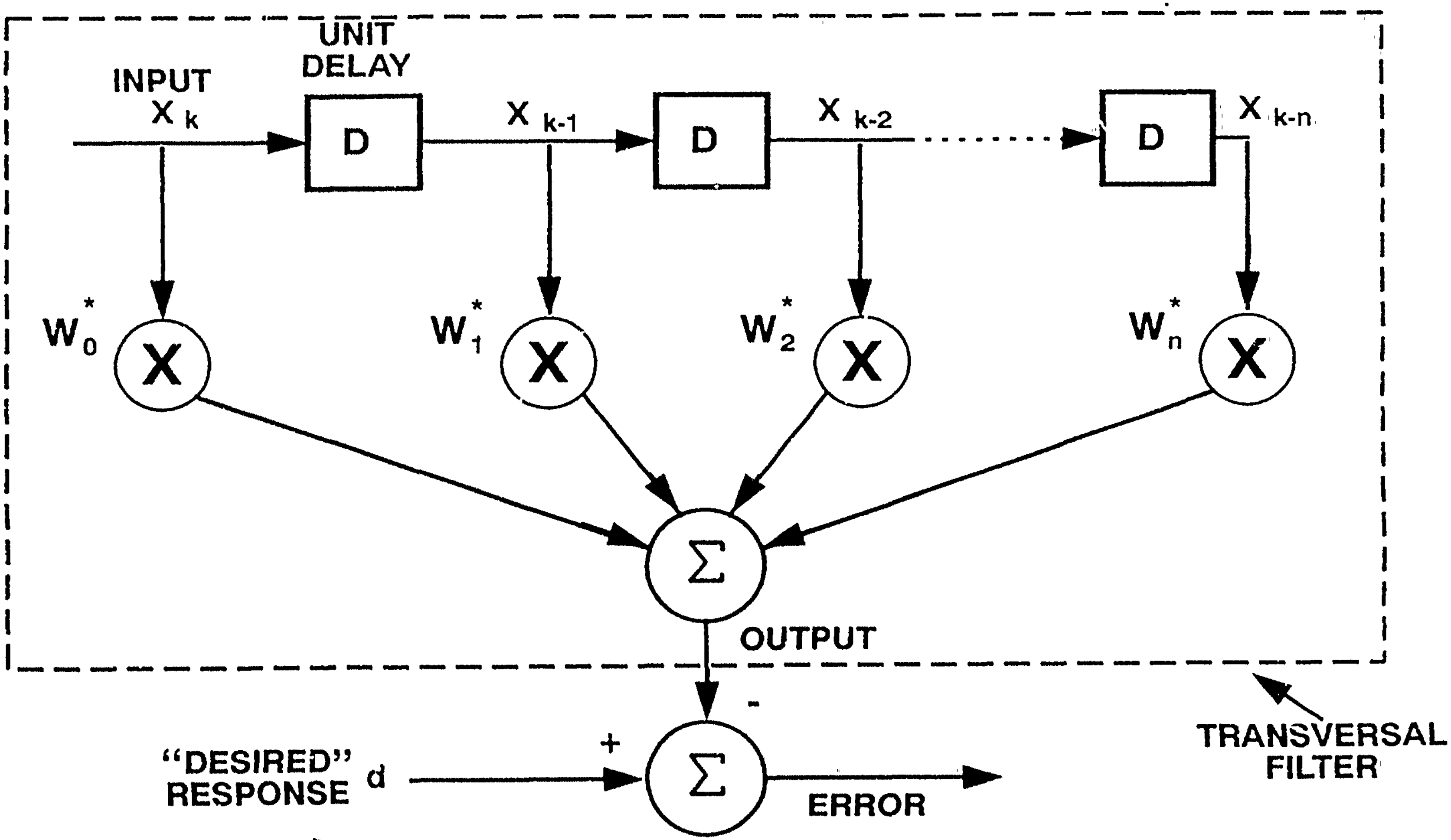
The receiver may not be able to provide the entire 130 dB of interference rejection filtering needed at the transmitter frequency. Therefore, an external applique capable of supplying the additional filtering may be required. An Adaptive Noise Canceler with a single input is one possible way of providing the additional filtering required.

Adaptive noise cancelers are not limited to separating narrowband signals that are close in frequency, i.e., they are not limited in application to just frequency hopping radios. A particular type of adaptive noise canceler known as an Adaptive Line Enhancer (ALE) is capable of separating narrow-band, deterministic signals from random wide-band signals (e.g., it is capable of protecting a weak wide-band, direct sequence spread spectrum signal from a strong, interfering, narrow-band signal).

Initially, the theoretical steady-state performance of both an adaptive noise canceler with a single input and an adaptive line enhancer will be described by assuming that the adaptive process has "converged" (i.e., the tap filter weights are no longer changing). These adaptive filters can then be approximated by and understood as Wiener filters.

A Wiener filter is essentially a transversal filter that produces an optimum output in a minimum mean square sense. A Wiener filter is shown in Figure 1. The output of a transversal filter is subtracted from a "desired" response, d, that is similar to but not exactly the same as the signal to be detected. The Wiener weights of the transversal filter are designed to minimize the mean square error = $E\left[\left(d - \sum_{i=0}^{n} W_i X_{k-i}\right)^2\right]$ at the output of the summer. When the Wiener weights are used, the transversal filter gives an optimum or best estimate of the true signal value (the signal that d, the desired response, is similar to).

In an effort to explain how an adaptive noise canceler with single input and an ALE actually work, the functional relationship between the optimal or Wiener PTF weight values (and hence the PTF frequency response) and the interfering and intended signal are developed in much more detail than is found in textbooks or



$\vec{W}^* = (W_0^*, W_1^*, W_2^*, \cdots W_n^*)$ = WIENER WEIGHT VECTOR

Figure 1. Wiener Filter

review articles. Building on this analytical foundation is then shown why:

1. For the case of a weak narrow-band intended signal versus a strong narrow-band interferer, the frequency response of the PTF within an adaptive noise canceler with single input is dominated or controlled by the strong interfering signal. This results in a PTF passband and an adaptive noise canceler notch around the interfering frequency.

2. For the case of either a weak random wide-band intended signal versus a strong narrow-band interferer or the case of a weak narrow-band intended signal versus a strong random wide-band interferer, the frequency response of the PTF in an ALE is determined by the narrow-band signal. This results in a PTF passband around the narrow-band frequency and a notch in the ALE output at this same narrow-band frequency.

After the steady-state performance of the subject adaptive filters has been described, three different adaptive algorithms (Differential Steepest Descent, Least Mean Square, and Random Search) are introduced. These algorithms describe how the adaptive filter tap weights must be iteratively modified in order to approach a "steady-state" condition.

Finally, a SAW device implementation of a PTF that could be used in building an adaptive noise canceler with single input or an ALE is described. Performance levels (maximum input power, interferences suppression, and switching speed) are given in order to illustrate its capabilities.

ADAPTIVE NOISE CANCELING

An Adaptive Noise Canceler as shown in Figure 2 works as follows:

"A signal is transmitted over a channel to a sensor that receives the signal plus an uncorrelated noise N_O. The combined signal and noise $S + N_O$ form the primary input to the canceler. A second sensor receives a noise N_1, which is uncorrelated with the signal but correlated in some unknown way with the noise N_O. This sensor provides the reference input to the canceler. The noise N_1 is filtered to produce an output Y that is a close replica of N_O. This output is subtracted from the primary input $S + N_O$ to produce the system output, $S + N_O - Y$."[1]

The output of the canceler is used to modify, via an appropriate adaptive algorithm, the frequency response of the adaptive filter.

The adaptive filter will usually be implemented as a programmable transversal filter (PTF) (see Figure 3). A transversal filter is the preferred implementation because:

1. It is one of the simplest filter structures. The filter output is simply the sum of delayed and scaled inputs.
2. There is no feedback from the taps to the input.
3. It is stable. Since there is no feedback, a finite filter input produces a finite filter output.
4. It has a linear phase characteristic, i.e., it produces a phase shift that is linearly proportional to frequency. It can be shown[19] that if a signal is to be passed through a linear system without any resultant distortion, the overall

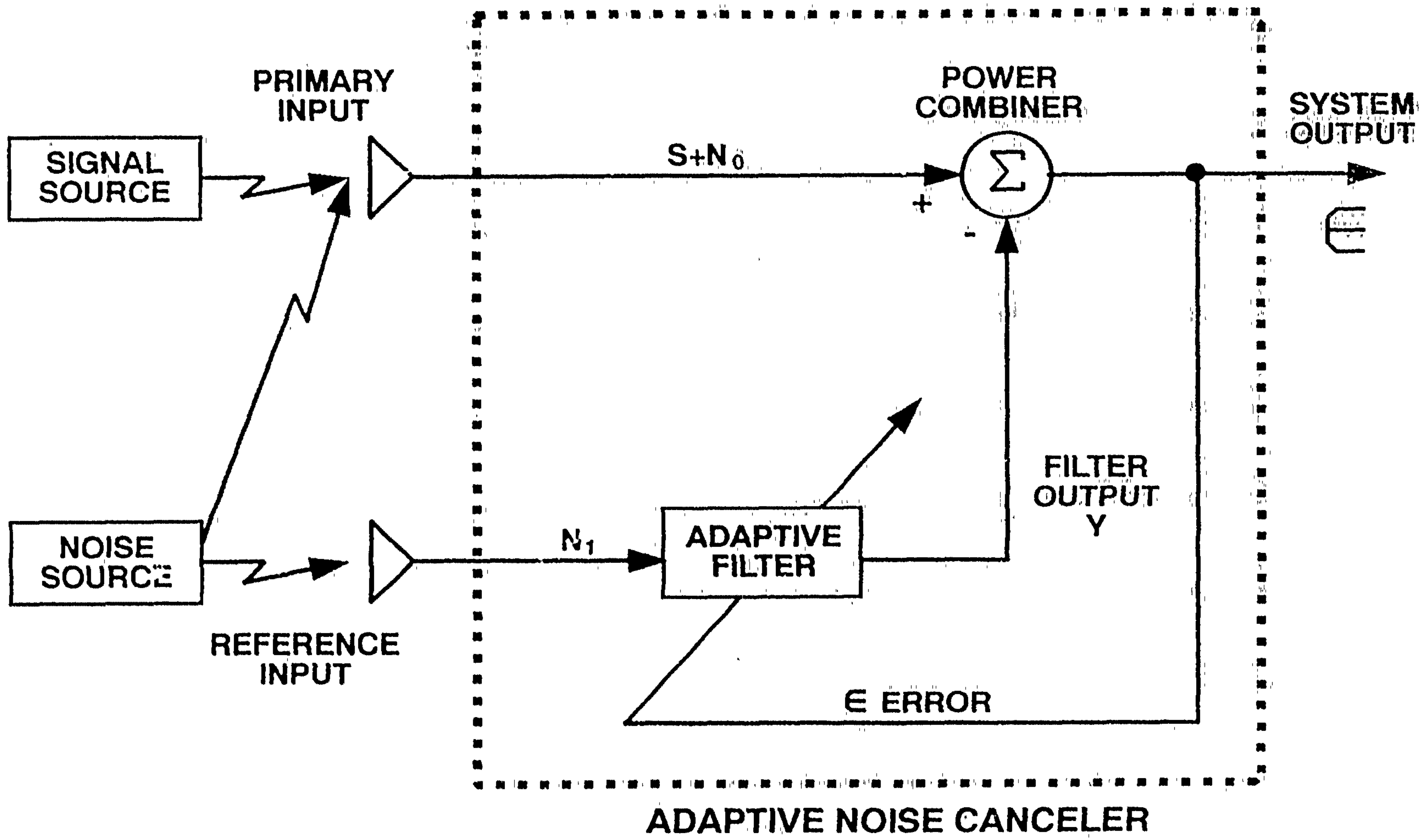


Figure 2. Adaptive Noise-Cancelling Concept (From Ref.8)

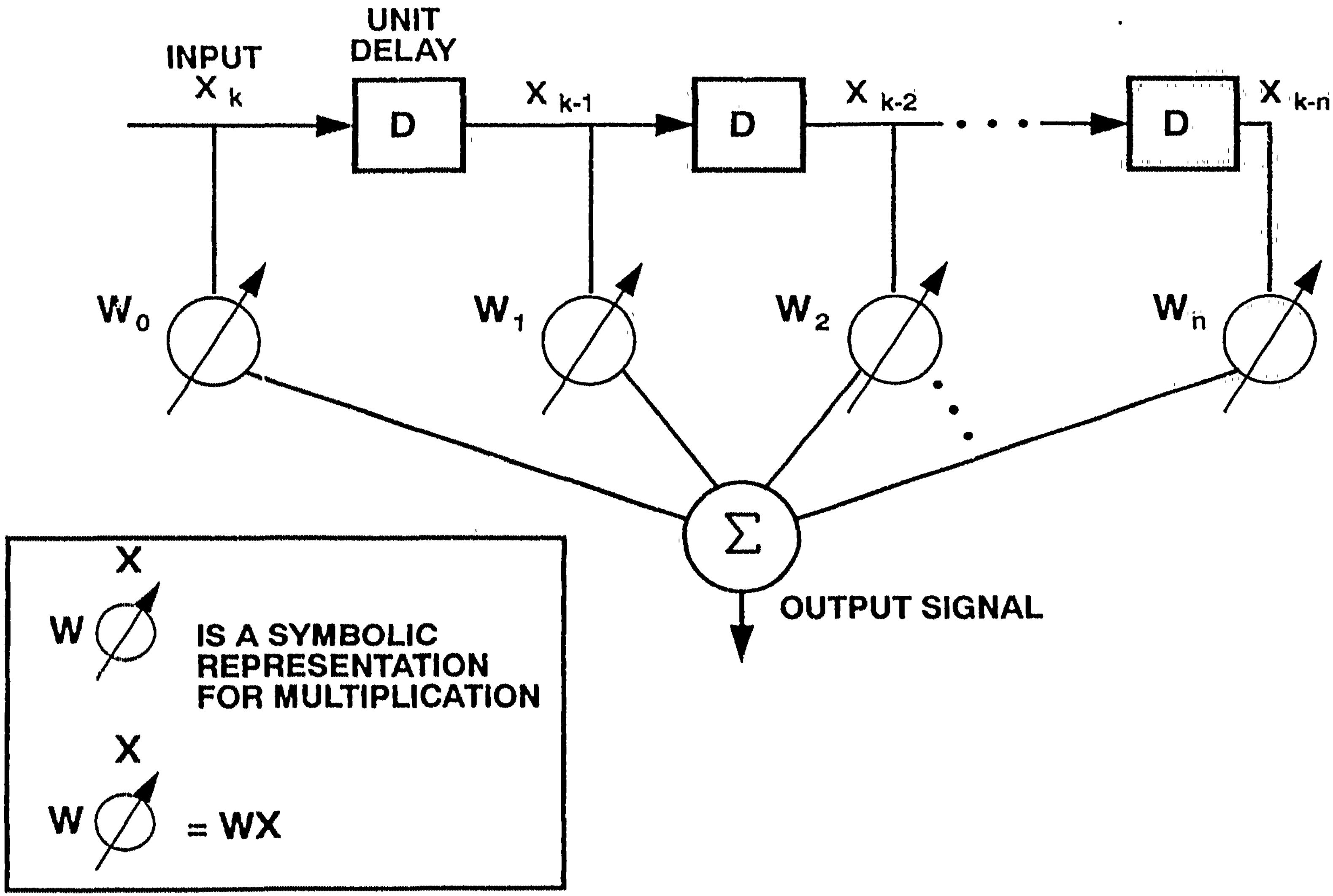


Figure 3. Programmable Transversal Filter (PTF)

system frequency response must have a constant amplitude gain characteristic over the frequency spectrum of the input signal and its phase shift must be linear over the same frequency spectrum. Filtering without distortion is important for adaptive noise canceling because the adaptive filter must pass the interference without distortion so that it can be subtracted (at the summer) from the unfiltered interferer. If the adaptive filter introduces distortion then the summer is no longer subtracting two identical interferers.

5. There is a simple and analytically tractable relationship between the frequency transfer function of a transversal filter and its parameters (see equation 47). The complicated nonlinear relationship between parameters and transfer function for most other filter structures makes the analysis and calculation of adaptive algorithms much more difficult than for transversal filters.

6. Widrow's algorithm, one of the most widely used adaptive algorithms, assumes a transversal filter structure.

A PTF forms a weighted sum of delayed versions of the input signal. It is programmable in that the weights can be changed. Changing the weights changes the frequency transfer function of the PTF. A PTF is identical in structure to a programmable finite impulse response (FIR) digital filter.

The specific technology used to implement a PTF will depend on the frequency range of interest. For VHF and UHF applications, Surface Acoustic Wave (SAW) devices are an appropriate technology.

At these frequencies, SAW technology can give the appropriate sampling rates (intertap delay) and total delay times necessary to implement transversal filters with the required frequency resolution needed for cosite interference reduction.

ADAPTIVE NOISE CANCELING WITH A SINGLE INPUT

Before an adaptive noise canceler can be implemented, a reference signal correlated with the interfering signal but not the intended signal must be generated. When the interfering signal N_o is much stronger than the intended signal S, the reference signal can be generated by modifying the adaptive noise canceler of Figure 2 to give the circuit shown in Figure 4. In Figure 4 the primary and reference inputs are connected together. In effect, Figure 4 assumes that the reference input is equal to the primary input. This may at first appear contradictory. The reference input N_1 has to be correlated to the interference N_o, not the signal S. But since the signal S is part of the primary input, it will be part of reference input if the reference input equals the primary input as per Figure 4. Hence, the reference input appears to be correlated to the signal also. When the interfering signal N_o is much larger than the intended signal ($N_o >> S$), the apparent contradiction is resolved. In this case the reference input N_1 ($N_1 = S + N_o$ = primary input) is highly correlated with and "looks" like the interfering signal N_o (i.e., $N_1 \approx N_o$).

While S is a component of N_1 and therefore will correlate to a certain extent with N_1, N_o is so much larger than S that N_1 will be much more highly correlated to N_o than S. So to a very good approximation, the reference input N_1 is correlated to the interference N_o not the signal S. This is what was to be proved.

It will now be shown why the reference input must be correlated to the interference and not the signal. The adaptive filter

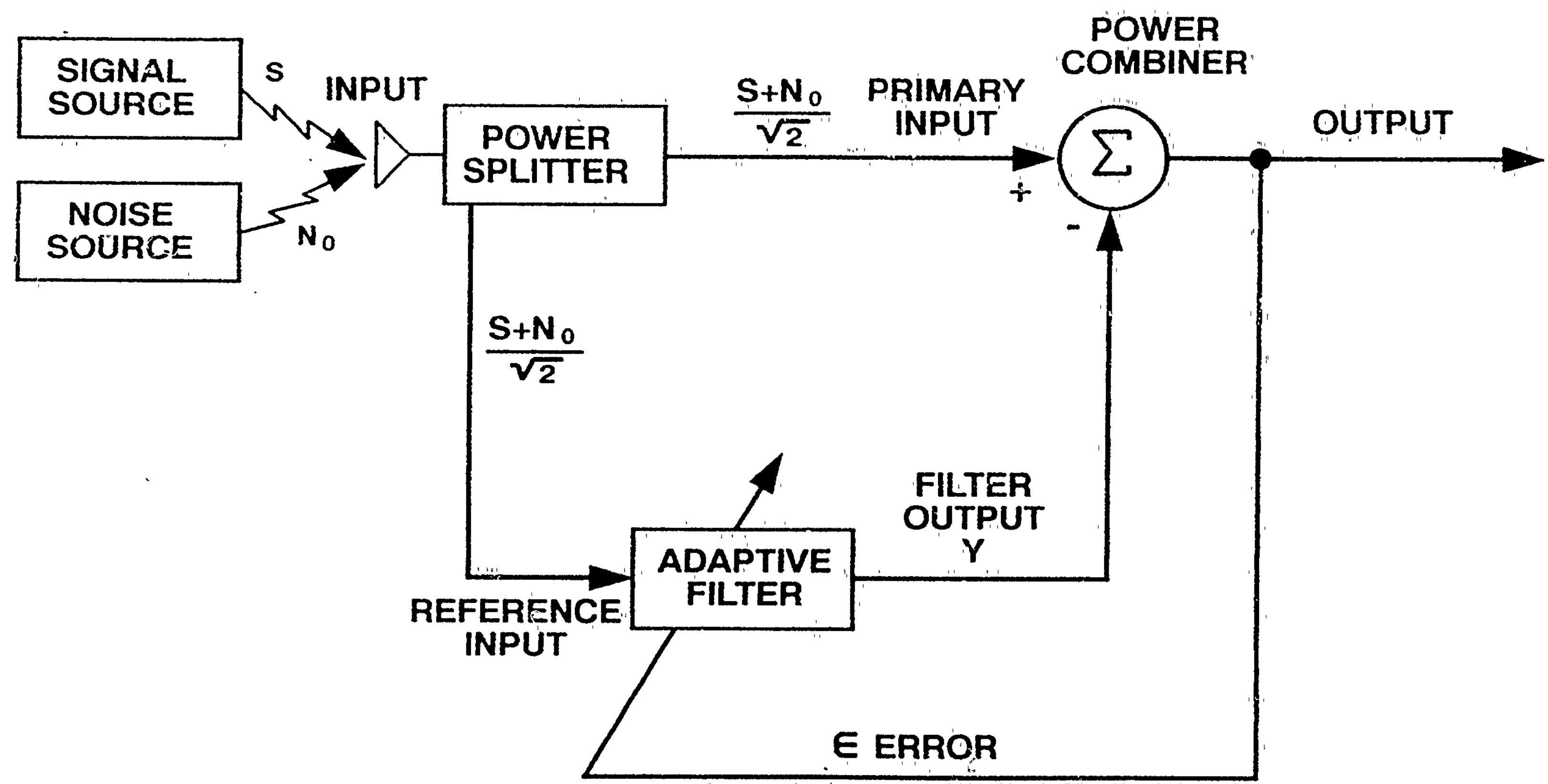


Figure 4. Adaptive Noise Canceler with a Single Input

within the canceler must filter the reference input N_1 to produce an output Y that is a close replica of N_0. If N_1 is not correlated to N_0, i.e., if N_1 does not "look" somewhat like N_0, then no amount of filtering can make Y look like N_0. To prove that the reference input (or primary input) of Figure 4 is more highly correlated to the interference than to the signal, first note than, the reference input equals

$$(S + N_0)/\sqrt{2}$$

where:

S = input signal amplitude

N_0 = input "noise" or interference amplitude

The factor $1/\sqrt{2}$ appears because the input power splitter is assumed to evenly split the power associated with the signal and interference amplitudes S and N_0. Since power is proportional to amplitude squared, reducing power by a factor of 2 means that amplitude is reduced by $\sqrt{2}$ at each output of the input power splitter.

Since we are assuming that N_0 is much larger than S, i.e., $N_0 >> S$, it follows that $(S+N_0)/\sqrt{2}$ is more highly correlated with N_0 than with S. To be more explicit, if we define[2] the average cross-correlation $R_{12}(\tau)$ between two waveforms $V_1(t)$ and $V_2(t)$ as

$$R_{12}(\tau) = \lim_{T\to\infty} \frac{1}{T} \int_{-T/2}^{T/2} V_1(t)\ V_2\ (t+\tau)dt \qquad (1)$$

where τ is the relative time displacement between the two waveforms V_1 and V_2. Then the correlation between the reference input and the noise input is

$$R_{(ref)(noise)}(\tau) = \lim_{T\to\infty} \frac{1}{T} \int_{-T/2}^{T/2} \left[\frac{S(t)+N_o(t)}{\sqrt{2}}\right] N_o(t+\tau)\,dt \tag{2}$$

The correlation between the reference input and the signal input is

$$R_{(ref)(noise)}(\tau) \quad \lim_{T\to\infty} \frac{1}{T} \int_{-T/2}^{T/2} \left[\frac{S(t)+N_o(t)}{\sqrt{2}}\right] S(t+\tau)\,dt \tag{3}$$

Since by assumption $N_O >> S$, at $\tau = 0$ the dominant term in the integrand of equation (2) for $R_{(Ref)(Noise)}(o)$ will be $(N_O(t))^2$ i.e., the limit of the integral can be approximated by

$$\lim_{T\to\infty} \frac{1}{T} \int_{-T/2}^{T/2} \frac{N_o(t))^2\,dt}{\sqrt{2}} \approx R_{(Ref)(Noise)}(o) \tag{4}$$

In a similar analysis, the dominant term in the integrand of equation (3) for $R_{(Ref)(Signal)}(o)$ will be $N_O(t)S(t)$. The limit of the integral can be approximated by

$$\lim_{T\to\infty} \frac{1}{T} \int_{-T/2}^{T/2} \frac{S(t)\cdot N_o(t)\,dt}{\sqrt{2}} \approx R_{(Ref)(Signal)}(o) \tag{5}$$

$N_O >> S$ implies that

$$(N_O(t))^2 \quad >> N_O(t)S(t) \tag{6}$$

Since $(N_O(t))^2/\sqrt{2}$ is the approximate integrand of $R_{(Ref)\ (Noise)}(o)$ and $N_O(t)\ S(t)/\sqrt{2}$ is the approximate integrand of

$R_{(Ref)(Noise)}(o)$,

equations (4) and (5) and inequality (6) imply that

$$R_{(Ref)(Noise)}(o) >> R_{(Ref)(Signal)}(o) \tag{7}$$

In other words inequality (7) indicates that the reference signal is much more highly correlated with the noise than with the signal, as was to be demonstrated. This means that the reference signal "looks" more like the interference, N_O, than the signal S.

As the adaptive algorithm iterates, it will cause the adaptive filter to form a bandpass around the interfering frequency, F_{N_O}. If the PTF has been properly designed, then the resulting bandpass filter will "pass" F_{N_O} the interfering frequency and "reject" the intended signal frequency. Then the output of the adaptive filter (the filtered reference signal) will "look" even more like $N_O/\sqrt{2}$ than the input signal. When this output is subtracted from $(S + N_O)/\sqrt{2}$, at the summer, a signal very similar to $S/\sqrt{2}$ will remain. The interference has been canceled. The circuit shown in Figure 3 does indeed behave as an adaptive noise canceler.

ADAPTIVE LINE ENHANCER

The discussion up to this point was only concerned about protecting a narrow-band signal from narrow-band interference. It is also desirable to be able to separate narrow-band signals from random broad-band signals. The waveforms encountered in communications systems are in many cases unpredictable. A random signal is often an appropriate model for a real signal. The following discussion will deal with separating both:

1. A weak random broad-band signal from a strong narrow-band interferer, and
2. A weak narrow-band signal from a strong random broad-band interferer.

An Adaptive Line Enhancer (ALE) illustrated in Figure 5 is one possible method of performing this signal separation. An adaptive line enhancer differs from an "Adaptive Noise Canceler with a single input" as shown in Figure 4 in that, a delay has been introduced preceding the adaptive filter. In order to understand how an ALE works, a more detailed analysis of Figure 4 will be necessary.

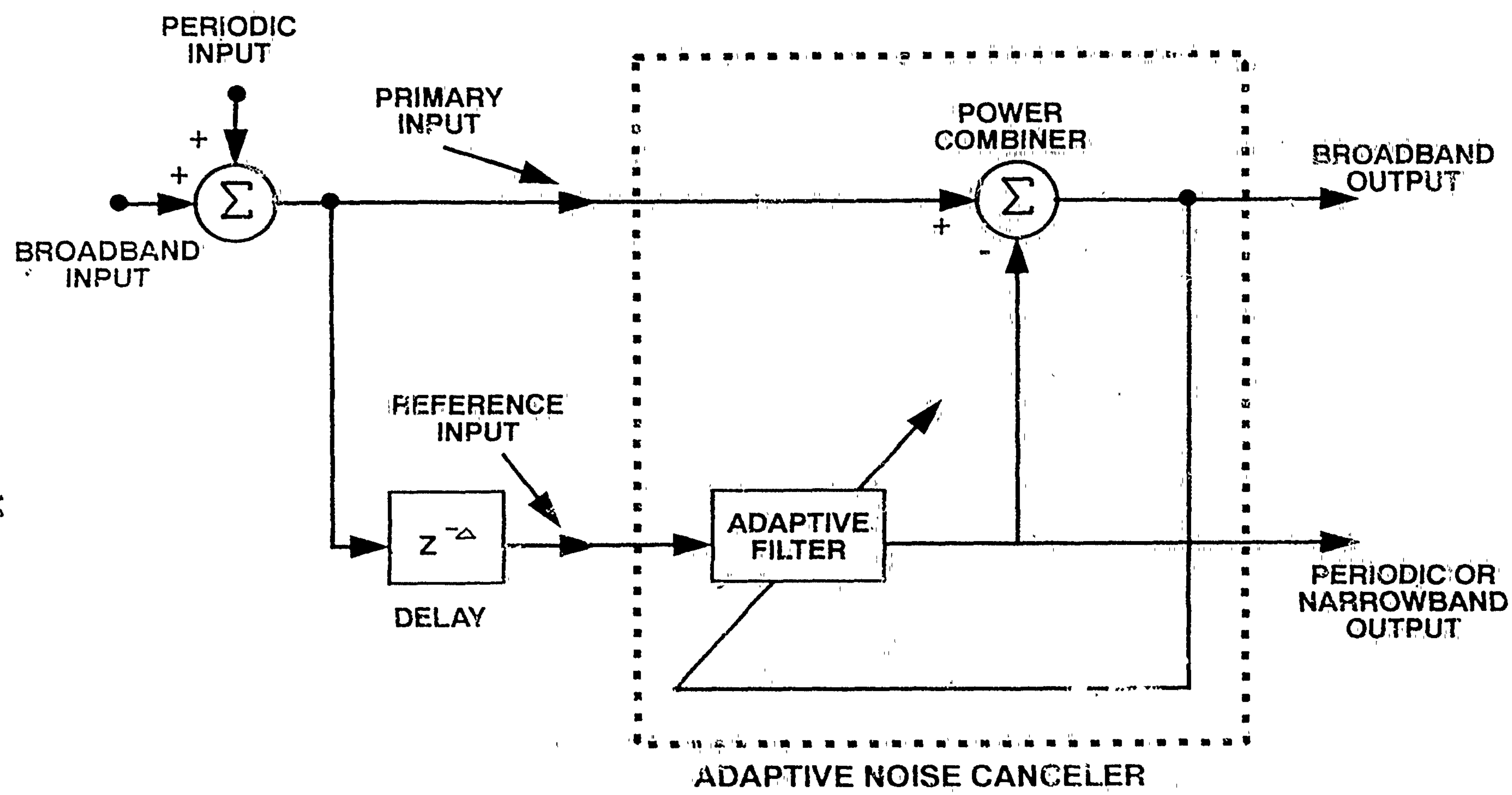


Figure 5. Adaptive Line Enhancer (From Ref. 8)

ANALYSIS OF AN ADAPTIVE NOISE CANCELER WITH A SINGLE INPUT

After the adaptive process has converged, the performance of the filter in the adaptive noise canceler of Figure 4 can be approximated by a Wiener filter. This means that after convergence the adaptive algorithm has produced (by adjusting the adaptive filter frequency response) a system output $((S+N_o)/\sqrt{2}) - Y$, that is a best fit in a minimum mean square error sense to $S/\sqrt{2}$. In other words, the mean square error is minimized, i.e., the average value taken over a large number of samples of,

$$\left(\text{system output} - \frac{\text{intended signal input}}{\sqrt{2}}\right)^2$$

$$= \left[\left(\frac{S+N_o}{\sqrt{2}} - Y\right) - \frac{S}{\sqrt{2}}\right]^2$$

is a minimum. In effect, the adaptive algorithm is minimizing the interference power at the adaptive noise canceler output by causing (via tap weight adjustment) the adaptive filter output Y to "look" like the interference N_o.

The adaptive filter frequency response can be controlled by varying its tap weights. The optimal weight vector W*, the Wiener weight vector, that minimizes the mean squared system output is given by

$$W^* = R^{-1}P \tag{8}$$

where R = Input Correlation Matrix

and

P = Cross-Correlation Column Vector

$$R = E \begin{bmatrix} X_k^2 & , X_k X_{k-1} & , X_k X_{k-2} & , \cdots, & X_k X_{k-n} \\ X_{k-1} X_k, & X_{k-1}^2 & , X_{k-1} X_{k-2}, & \cdots, & X_{k-1} X_{k-n} \\ \vdots & & & & \\ X_{k-n} X_k, & X_{k-n} X_{k-1}, & X_{k-n} X_{k-2}, & \cdots, & X_{k-n}^2 \end{bmatrix} \qquad (9)$$

The symbol E means that the matrix R is composed of the expected or mean values of the indicated products of adaptive filter tap outputs (see Figure 3). The main diagonal terms of R are the mean squares of the tap outputs. The off-diagonal terms are the cross-correlation among the tap outputs.

$$P = E\, [\, d_k X_k,\ d_k X_{k-1},\ d_k X_{k-2},\ \cdots,\ d_k X_{k-n}\,] \qquad (10)$$

where d_k is the desired response at "time" k. When X_k is the reference input to the adaptive noise canceler of Figure 4, d_k is the primary input. In terms of Figure 4's notation:

$$d_k = \frac{S + N_o}{\sqrt{2}} \qquad (11)$$

$$X_k = \frac{S + N_o}{\sqrt{2}} \qquad (12)$$

The components of the vector P are the cross-correlations between the desired response and the adaptive filter tap outputs.

Equations 8, 9, and 10 can be used to investigate the influence of the interferer and the intended signal on the optimal weight vector W*. Of particular interest are those conditions under which W* and hence the frequency response of the adaptive filter are only a function of the interfering signal. This is what will allow the adaptive filter to form a "bandpass" around the interferer and reject the intended signal.

A typical element of the autocorrelation matrix (equation 9) is $E\,[X_{k-i} \cdot X_{k-j}]$, i.e., $R_{ij} = E\,[X_{k-i} \cdot X_{k-j}]$

where:

X_k = signal input to the adaptive filter at time k or at sample k.

X_{k-i} = total signal at the ith tap of the adaptive filter.

X_{k-j} = total signal at the jth tap of the adaptive filter.

k is a time index, not necessarily a unit of time.

If we assume, as per Figure 4, that the input to the adaptive filter is

$$X_k = \frac{S + N_o}{\sqrt{2}}$$

where:

S = signal

N_o = noise or interference

then

$$E\,[X_{k-i} \cdot X_{k-j}] = 1/2\ E\,[(S+N_o)_{k-i} \cdot (S+N_o)_{k-j}] \tag{13}$$

$$= 1/2\ E\,[(S_{k-i} + N_{o_{k-i}}) \cdot (S_{k-j} + N_{o_{k-j}})]$$

$$E\,[X_{k-i} \cdot X_{k-j}] = 1/2\ (E\,[(S_{k-i} \cdot S_{k-j}) + (S_i \cdot N_{o_{k-j}}) + (N_{o_{k-i}} \cdot S_{k-j}) + (N_{o_{k-i}} \cdot N_{o_{k-j}})] \tag{14}$$

$$E\,[X_{k-i} \cdot X_{k-j}] = 1/2\ (E\,[S_{k-i} \cdot S_{k-j}] + E\,[S_{k-i} \cdot N_{o_{k-j}}] + E\,[N_{o_{k-i}} \cdot S_{k-j} + E\,[N_{o_{k-i}} \cdot N_{o_{k-j}}]) \tag{15}$$

Interference occurs when the noise is much larger than the intended signal, i.e., $N_o >> S$. We shall therefore assume that:

$N_o >> S$ (16)

The last term in equation 15 which is a function of the interference but not the intended signal will usually be the largest term in the equation since all other terms are expected values of products containing S (the intended signal). Clearly if the interference is greater than the signal ($N_o >> S$), then $N_o^2 > N_o S > S^2$ and in most cases

$$E\,[N_{o_{k-i}} \cdot N_{o_{k-j}}]$$

will be larger than either

$$E\,[S_{k-i} \cdot S_{k-j}],$$

$$E\,[S_{k-i} \cdot N_{o_{k-j}}], \text{ or}$$

$$E\,[N_{o_{k-i}} \cdot S_{k-j}].$$

It is possible for $N_{o_{k-i}}$ and $N_{o_{k-j}}$ to be 90 degrees out of phase (for narrow-band deterministic interference). In this case, $E(N_{o_{k-i}} \cdot N_{o_{k-j}})$ might not be larger than the other terms in equation 15 and the sum of all four terms would be of order N_oS which is much smaller than N_o^2. Every element of the autocorrelation matrix R is either dominated by the interference N_o or is small compared to it. If the autocorrelation matrix R is dominated by the interference, it can be shown that R^{-1} will also be dominated by the interference.

The Wiener weight vector W* that minimizes the mean square adaptive noise canceler output is given by equation 8. The preceding analysis has shown that R and hence R^{-1} are dominated by or are primarily functions of the interfering signal. If it can be shown that P the cross-correlation column vector is also dominated by the interfering signal, then equation 8 will imply that the Wiener weight vector W* is primarily a function of the interference. As mentioned previously, this primary dependence of W* on the interfering signal is what will allow the adaptive filter to form a bandpass around the interferer: pass the interferer and reject the intended signal.

The cross-correlation vector P will now be investigated. From equations 10, 11, and 12:

$$P = E\,[d_k X_k,\ d_k X_{k-1},\ d_k X_{k-2},\ \cdots,\ d_k X_{k-n}] \tag{10}$$

$$d_k = \frac{S_k + N_{o_k}}{\sqrt{2}} \tag{11}$$

$$X_k = \frac{S_k + N_{o_k}}{\sqrt{2}} \tag{12}$$

also

$$X_{k-i} = \frac{S_{k-i} + N_{o_{k-i}}}{\sqrt{2}} \tag{17}$$

A typical element of P is:

$$P_i = E\,[d_k \cdot X_{k-i}] \tag{18}$$

where i can vary between 0 and n.

Substituting equations 11 and 12 into equation 18 gives:

$$P_i = 1/2\ E\,[d_k \cdot X_{k-i}] = E\,[(S_k + N_{o_k})\cdot(S_{k-i} + N_{o_{k-i}})] \tag{19}$$

$$= 1/2\ E\,[(S_k\, S_{k-i}) + (S_k\, N_{o_{k-i}}) + (N_{o_k}\, S_{k-i}) + (N_{o_k}\, N_{o_{k-i}})]$$

$$P_i = 1/2\ (E\,[S_k S_{k-i}] + E\,[S_k N_{o_{k-i}}] + E[N_o S_{k-i}] + E[N_{o_k}\, N_{o_{k-i}}]) \tag{20}$$

The analysis of equation 20 is now very similar to the analysis of equation 15. Since it is assumed that $N_O >> S$, the last term of the equation,

$$E\,[N_{O_k} \cdot N_{O_{k-i}}]$$

will usually be the largest term of the equation because all the other terms are expected values of products containing S. Eventually the same conclusion will be reached about the cross-correlation vector P that was arrived at in reference to the autocorrelation matrix R, that is, every element of P is either dominated by N_O or small compared to it.

Since P and R^{-1} are dominated by N_O, it follows from equation 8 that W*, the optimal weight vector, will also be dominated by the interference. The inteference "controls" the optimal weights. This is the conclusion that was to be established.

The adaptive noise canceler circuit of Figure 4 works when both the intended signal and the interferer are narrow-band. When the strong interfering input to the circuit is a random wide-band signal, the canceler will not be able to filter it out. The PTF will not be able to reject the weak narrow-band intended signal and pass the random wide-band interferer (assuming they overlap in frequency) as was done in the narrow-band interferer vs. narrow-band intended signal case previously discussed.

If the PTF could put a passband around the narrow-band intended signal and filter out most of the strong random wide-band interferer, then signal separation could be achieved. In effect, this means that the narrow-band intended signal would control the PTF frequency response as opposed to the narrow-band interferer vs. narrow-band intended signal case where the interference dominated and controlled t . PTF frequency response.

If an appropriate delay is placed in front of the PTF in the adaptive noise canceler, as shown in Figure 5, the resulting circuit is known as an Adaptive Line Enhancer (ALE). This circuit is capable of putting a passband around a narrow-band intended signal in the presence of a strong wide-band random signal. As a result, it is capable of separating these two types of signals. In the following section it will be shown why an ALE works this way.

ANALYSIS OF AN ADAPTIVE LINE ENHANCER

Equation 8, will be used to analyze the adaptive line enhancer shown in Figure 5. The analysis will show that the ALE can be used to separate the following:

CASE 1 - A weak, random broad-band signal from a strong narrow-band interferer.

CASE 2 - A weak, narrow-band signal from a strong random, broad-band interferer.

For both cases:

$S \equiv$ weak intended signal

$N_o \equiv$ strong interferer or "noise" where $N_o >> S$

CASE 1

S = weak, random, broad-band signal

N_o = strong, narrow-band interferer

Equation 10 for P the cross-correlation vector has as its components the cross-correlations between the desired response (d_k) and the adaptive filter tap outputs (X_k, $X_{k-1}, \ldots,$ X_{k-n}). d_k is the input to the positive terminal of the second or output summer as shown in Figure 5.

Assuming that the input power is evenly split between the primary and reference (upper and lower) branches of the ALE circuit:

$$d_k = \frac{S_k + N_{o_k}}{\sqrt{2}} \tag{21}$$

Where S_k and N_{o_k} indicate that each of these signals is sampled at the time corresponding to time index k.

The amplitude that will be the input to the delay element in the lower branch of the ALE is also $(S_k + N_{o_k})/\sqrt{2}$. The delayed output is denoted by $(DS_k + DN_{o_k})/\sqrt{2}$, where "D" indicates that the signal has been delayed by delta (Δ) units of time. Thus $(DS_k + DN_{o_k})/\sqrt{2}$ is the input to the adaptive filter, i.e.,

$$X_k = (DS_k + DN_{o_k}) \ / \ \sqrt{2} \tag{22}$$

The signal on the first tap of the adaptive filter is

$$X_{k-1} = (DS_{k-1} + DN_{o_{k-1}}) \ / \ \sqrt{2} \tag{23}$$

This means that the signal out of the first tap introduces a time delay of one sample period, i.e., S_{k-1} and $N_{o_{k-1}}$ denote S_k and N_{o_k} delayed by one sample period. The signal amplitude on the i<u>th</u> tap is X_{k-i}

$$X_{k-i} = (DS_{k-i} + DN_{o_{k-i}}) \ / \ \sqrt{2} \tag{24}$$

A typical component of the cross-correlation vector P, such as the i<u>th</u> component, is $E\,[d_k X_{k-i}]$. Equations 21 and 24 for d_k and X_{k-i} imply that:

$$E[d_k X_{k-i}] = E\left[\frac{(S_k+N_{o_k})}{\sqrt{2}} \cdot \frac{(DS_{k-i}+DN_{o_{k-i}})}{\sqrt{2}}\right] \tag{25}$$

$$= 1/2\ E[S_k \cdot DS_{k-i} + S_k \cdot DN_{o_{k-i}} + N_{o_k} \cdot DS_{k-i} + N_{o_k} \cdot DN_{o_{k-i}}]$$

$$E[d_k \cdot X_{k-i}] = 1/2\ (E[S_k \cdot DS_{k-i}] + E[S_k \cdot DN_{o_{k-i}}] + E[N_{o_k} \cdot DS_{k-i}] + E[N_{o_k} \cdot DN_{o_{k-i}}]) \tag{26}$$

The purpose of introducing a delay element into an adaptive noise canceler to form an ALE is to decorrelate the wide-band component of the input from itself. If the delay time delta (Δ) is chosen larger than the autocorrelation time of the wide-band signal, then the correlation between the delayed and the original wide-band component will be zero by definition of autocorrelation time. For Case 1, S is the weak random broad-band intended signal. If the delay time Δ is larger than the autocorrelation time of S, then:

$$E[S_k \cdot DS_{k-i}] = 0 \tag{27}$$

for all i or equivalently for all taps of the adaptive filter. The left side of equation 27 is just the first term of equation 26.

The analysis now becomes very similar to the analysis of the cross-correlation vector of an adaptive noise canceler. Since it is assumed that the noise or narrow-band interference N_o is much larger than the signal S ($N_o >> S$), this implies that in most cases the last term of equation 26 will be much larger than either of the other two non-zero terms, i.e.,

$$E[N_{o_k} \cdot DN_{o_{k-i}}] > E[S_k \cdot DN_{o_{k-i}}] \tag{28}$$

and

$$E\,[N_{o_k} \cdot DN_{o_{k-i}}] > E\,[N_{o_k} \cdot DS_{k-i}] \tag{29}$$

It is possible, however that N_o and $DN_{o_{k-i}}$ may be 90 degrees out of phase. In this case,

$$E\,[N_{o_k} \cdot DN_{o_{k-i}}]$$

might not be larger than the other terms and inequalities 28 and 29 would not be valid. But then the sum of all three non-zero terms would be of order N_oS, which is much smaller than $N_o{}^2$. Therefore, every component of the cross-correlation vector P is either dominated by the narrow-band interference N_o, via inequalities 28 and 29 and equation 26 or is small compared to N_o.

A typical element of the autocorrelation matrix for an ALE is

$$E\,[X_{k-i} \cdot X_{k-j}] = 1/2\ E\,[(DS_{k-i} + DN_{o_{k-i}}) \cdot (DS_{k-j} + DN_{o_{k-i}})] \tag{30}$$

$$\begin{aligned} E\,[X_{k-i} \cdot X_{k-j}] = 1/2\ (&E\,[DS_{k-i} \cdot DS_{k-j}] + \\ &E\,[DS_{k-i} \cdot DN_{o_{k-j}}] + E\,[DN_{o_{k-i}} \cdot DS_{k-j}] + \\ &E\,[DN_{o_{k-i}} \cdot DN_{o_{k-j}}]) \end{aligned} \tag{31}$$

Equation 31 is very similar to equation 15 for a typical element of the autocorrelation function of an adaptive noise canceler with a single input. The only difference is the delay. The analysis of equation 31 is exactly the same as equation 15. Since the interference N_o is much larger than the intended signal, every element of the autocorrelation matrix R for an ALE is either dominated by the narrow-band interference or is small compared to it. This will also be true for the inverse, R^{-1}. It was previously shown that this is also true for the ALE cross-correlation matrix P. Therefore equation 8 for the optimal weight vector $W^* = R^{-1}P$ implies that the weight vector that the adaptive filter "converges" to is primarily a function of the interfering narrow-band signal, N_O.

This is why the adaptive filter in an ALE puts a "bandpass" around the interferer. For all practical purposes it never "sees" (via equations 8, 9, and 10 for W^*, R^{-1} and P, respectively) the intended random wide-band signal.

In other words, for Case 1, (a weak random broad-band signal and a strong narrow-band interferer) it has been shown that R and P (given by equations 9 and 10, respectively) are primarily functions of, or are dominated by N_O. Equation 8 then implies that the optimum weight vector W^* is dominated by N_O. Equation 47 (see Case 2 analysis) gives the frequency response $H(\omega)$ of the PTF as:

$$H(\omega) = \sum_{i=1}^{n} W_i \; e^{j\omega\Delta(-i)} \tag{47}$$

where:

$H(\omega)$ = frequency transfer function

ω = frequency

Δ = intertap delay

n = Number of taps

$j = \sqrt{-1}$

The frequency response $H(\omega)$ is a function of the weights W_i. The optimum weight vector W^* is primarily a function of N_O the narrow-band interferer. A consequence of the domination of W^* by N_O is that when W^* is substituted into equation 47, $H(\omega)$ develops a peak or maxima around the frequency of the narrow-band signal. It is in this sense that the PTF frequency response never "sees" the intended weak random broad-band signal.

CASE 2

Let S = weak narrow-band intended signal

N_o = strong wide-band random interferer

Equation 8, $W^* = R^{-1} P$, was again used to analyze the ALE. The ith component of the cross-correlation vector P is still given by equation 26. Now N_o, the strong interferer, is a wide-band random signal. It is again assumed that the delay time Δ is chosen larger than the autocorrelation time of the wide-band random signal. As a result, correlation between the delayed and original wide-band random signal will be zero, i.e.,

$$E\,[N_{o_k} \cdot DN_{o_{k-i}}] = 0 \tag{32}$$

Thus, for case 2, $E\,[N_{o_k} \cdot DN_{o_{k-i}}]$ is not the dominant term in equation 26 that it was for case 1 and in fact it makes no contribution to equation 26.

Thus, by the introduction of an appropriate delay time Δ, the influence that $E\,[N_{o_k} \cdot N_{o_{k-i}}]$ had in equation 20 for the cross-correlation matrix element for an adaptive noise canceler with single input becomes nullified. Since the interference N_o is assumed to be much larger than the intended signal S,

$$E\,[N_{o_k} \cdot N_{o_{k-i}}]$$

for the adaptive noise canceler with single input or

$$E\,[N_{o_k} \cdot DN_{o_{k-i}}]$$

for an ALE has the potential to be the dominant term in equation 20 or 26, respectively. The elimination of the left-hand side of equation 32 is the major effect that the time delay in the ALE produces.

The interferer can only contribute to the cross-correlation element via the second and third terms of equation 26,

$E\ [S_k \cdot DN_{o_{k-i}}]$ and $E\ [N_{o_k} \cdot DS_{k-i}]$.

However, since $N_o >> S$, these terms will be many orders of magnitude smaller than $E\ [N_{o_{k-i}} \cdot N_{o_{k-i}}]$ or $E\ [N_{o_k} \cdot DN_{o_{k-i}}]$, where the intertap delay Δ is not chosen long enough to decorrelate N_o. In the ideal case, if there is no correlation between the signal S and the interference, then both $E\ [S_k \cdot DN_{o_{k-i}}]$ and $E\ [N_{o_k} \cdot DS_{k-i}]$ will equal zero. Then in equation 26 only the first term, $E\ [S_k \cdot DS_{k-i}]$, will be non-zero. This term is a function of only the intended signal, not the interference. So if a weak narrow-band intended signal and a strong wide-band random interference are uncorrelated, the cross-correlation vector is only a function of the intended signal, not the interference.

Thus, for an ALE an appropriate time delay will minimize the effect of the wide-band random interference on the cross-correlation vector P. Since $W* = R^{-1}P$, it is necessary to know how interference and the time delay affect R, the autocorrelation matrix and its inverse R^{-1}. A typical element of the autocorrelation matrix for an ALE is given by equations 30 and 31. The last term in equation 31 is potentially the largest term since $N_o >> S$. This term gives the major effect of the interfering signal on the autocorrelation matrix.

The conditions under which the last term in equation 31, $E\ [DN_{o_{k-i}} \cdot DN_{o_{k-j}}]$ is zero or relatively small will now be investigated.

Assume that the random wide-band interference is white noise, i.e., with a power spectral density that is constant (say C) for all frequencies. It can be shown[3] that the Fourier transform of the power spectral density of this white noise,the autocorrelation function R (τ), is the same constant times a delta function

$$\delta(\tau) \text{ i.e. } R(\tau) = C\delta(\tau). \tag{33}$$

Equation 33 implies that $R(\tau)$ is equal to zero except for $\tau = 0$. This means that for a white noise signal N(t), N(t) and $N(t + \tau)$ are uncorrelated and independent no matter how small τ becomes.

The fourth term of equation 31, $E\,[DN_{o_{k-i}} \cdot DN_{o_{k-j}}]$, is basically the autocorrelation of the delayed interference input (DN_{o_k}) to the adaptive filter of the ALE. The correlation is performed between the ith and jth taps of the filter. It correlates the interference output that appears at the ith and jth taps using a correlation delay that is equal to the propagation delay between the two taps.

If it is assumed that N_o is white noise, then equation 33 implies that

$$E\,[DN_{o_{k-i}} \cdot DN_{o_{k-j}}] = 0 \qquad \text{when} \quad i \neq j \tag{34}$$

and that

$$E\,[DN_{o_{k-i}} \cdot DN_{o_{k-j}}] = C \qquad \text{when} \quad i = j \tag{35}$$

If it is further assumed that the signal S and the interference N_o are uncorrelated, then the second and third terms of equation 31 are zero for all values of i and j.

Thus, for white noise interference, the autocorrelation matrix given by equation 31 is as follows:

for the off diagonal elements $(i \neq j)$ equation 34 implies that:

$$R_{ij} = 1/2\ E\ [DS_{k-i} \cdot DS_{k-j}] \tag{36}$$

for the diagonal elements (i=j):

$$R_{ii} \equiv E\ [X_{k-i} \cdot X_{k-i}] = 1/2\ (E\ [DS_{k-i} \cdot DS_{k-i}] + E\ [DN_{o_{k-i}} \cdot DN_{o_{k-i}}]) \tag{37}$$

Substituting equation 35 into equation 37 implies:

$$R_{ii} = 1/2\ (\ E\ [DS_{k-i} \cdot DS_{k-i}] + C) \tag{38}$$

and since $N_o >> S$,

$$E\ [DN_{o_{k-i}} \cdot DN_{o_{k-i}}] >> E\ [DS_{k-i} \cdot DS_{k-i}] \tag{39}$$

Inequality 39 when substituted into either equation 37 or 38 implies that

$$R_{ii} = C/2\ \text{(APPROXIMATELY)} \tag{40}$$

It follows from inequality 39 and equation 40 that the off diagonal elements (given by equation 36) are small compared to the diagonal elements. Expressed as an inequality;

$$R_{ii} >> R_{ij} \tag{41}$$

Inequality 41 and equation 40 imply that for white noise interference, the autocorrelation matrix R can be approximated by a matrix that is both diagonal and scalar (a scalar matrix is a diagonal matrix whose diagonal elements are all equal)

$$R = \begin{bmatrix} C/2, 0, \ldots\ldots, 0 \\ 0, C/2, 0, \ldots, 0 \\ \vdots \quad \ddots \quad \vdots \\ 0 \ \ldots \quad C/2 \end{bmatrix} \tag{42}$$

If a matrix is scalar, its inverse will also be scalar. So R^{-1} can be expressed as follows:

$$R^{-1} = \begin{bmatrix} K, 0, \ldots\ldots\ldots, 0 \\ 0, K, 0, \ldots\ldots, 0 \\ \vdots \quad \ddots \quad \vdots \\ 0 \ldots \qquad K \end{bmatrix} \tag{43}$$

where K is some function of C, the power spectral density of the wide-band random interferer, K can be factored out of equation 43 to give:

$$R^{-1} = K \begin{bmatrix} 1, 0, \ldots\ldots\ldots, 0 \\ 0, 1, 0, \ldots\ldots, 0 \\ \vdots \quad \ddots \quad \vdots \\ 0 \ldots \qquad 1 \end{bmatrix} \tag{44}$$

The matrix in equation 44 is the identity matrix I. Equation 44 now becomes:

$$R^{-1} = KI \tag{45}$$

Where K is a scalar or number not a matrix.

Substituting equation 45 into equation 8, $W^* = R^{-1} P$, for the optimal weight vector of the adaptive filter gives

$$W^* = KIP = KP \tag{46}$$

since IP=P.

Equation 46 can be used to investigate the frequency transfer function of the adaptive filter. The adaptive filter is a tapped delay line or transversal filter. The frequency response of a tapped delay can be shown[4] to be

$$H(\omega) = \sum_{i=1}^{n} W_i e^{j\omega\Delta(-i)} \tag{47}$$

where:

$H(\omega)$ = frequency transfer function

ω = frequency

Δ = intertap delay

n = number of taps

$j = \sqrt{-1}$

Substituting equation 46 into equation 47 gives:

$$H(\omega) = \sum_{i=1}^{n} W_i e^{-j\omega\Delta i} = \sum_{i=1}^{n} KP_i e^{-j\omega\Delta i} \tag{48}$$

$$H(\omega) = K \sum_{i=1}^{n} P_i e^{-j\omega\Delta i} \tag{49}$$

Equation 49 indicates that K (and hence C) does not affect the relative frequency response, i.e.,

$$\frac{H\ (\omega_1)}{H\ (\omega_2)} = \frac{K \sum_{i=1}^{n} P_i e^{-j(\omega_1)\Delta i}}{K \sum_{i=1}^{n} P_i e^{-j(\omega_2)\Delta i}} = \frac{\sum_{i=1}^{n} P_i e^{-j(\omega_1)\Delta i}}{\sum_{i=1}^{n} P_i e^{-j(\omega_2)\Delta i}} \tag{50}$$

K is just a multiplicative or scale factor in equation 49. It cannot affect the relative frequency response, $H(\omega_1)/H(\omega_2)$, because it cancels out in equation 50. Thus, for a white noise interferer uncorrelated with the signal, the use of the optimal weights W* for the adaptive filter in the ALE causes the relative frequency response to be determined by the cross-correlation vector P. However, it was previously shown that for an ALE, an appropriate time delay will minimize or possibly eliminate the effect of the wide-band random interference on the cross-correlation vector P. P will be determined by S, the weak narrow-band signal (assuming that the signal S and the interference n_o are uncorrelated). The signal S will determine the relative frequency response (via equation 50), i.e., S will determine the frequency response up to a scale factor. The interferer N_o will determine the scale factor K (K is a function of C the power spectral density of N_o).

Therefore, for white noise interference, it is the weak narrow-band intended signal that determines what frequencies are passed or rejected by the adaptive filter. This is why the adaptive filter (for Case 2) can put a "bandpass" around the signal S and later subtract it from S + N_o at the summer.

The key assumption in the above analysis was that the wide-band random interferer was white noise. White noise uncorrelated with the signal implies that R (via equation 42) and R^{-1} (via equations 43 and 44) are scalar matrices. The scalar matrix R^{-1} implies equation 46: W* = KP. Equation 46 implies that the relative frequency response is determined by P. But the correlation vector P is determined by the signal S. Thus it was concluded that the relative frequency response is determined by the intended narrow-band signal.

It will now be determined whether or not the conclusion, that the relative frequency response of the adaptive filter is determined by the intended narrow-band signal, is still valid if the wide-band random interferer is not white noise. If R^1 and R^{-1} still remain scalar matrices, then the conclusion will remain valid. A typical element of the autocorrelation matrix R for an ALE is given by equation 31. If the noise and the signal are uncorrelated equation 31 becomes:

$$R_{ij} = 1/2\ (E\ [DS_{k-i} \cdot DS_{k-j}] + E\ [DN_{o_{k-i}} \cdot DN_{o_{k-j}}]) \qquad (51)$$

If it is assumed that $N_o >> S$ then the diagonal terms of equation 51 are given by

$$R_{ii} = 1/2\ E\ [DN_{o_{k-i}} \cdot DN_{o_{k-i}}] \qquad (52)$$

R_{ii} is a measure of the energy at tap i of the adaptive filter. Assuming that the same energy appears at each tap, then R_{ii} will have the same value for all i, i.e.,

$$R_{11} = R_{22} = \ldots = R_{NN} \qquad (53)$$

The off-diagonal elements of R are still given by equation 51 since $N_o >> S$. The first terms of equation 51 will be small compared to the diagonal elements, i.e.,

$$E\ [DN_{o_{k-i}} \cdot DN_{o_{k-i}}] >> E\ [DS_{k-i} \cdot DS_{k-j}] \qquad (54)$$

If the second term of equation 51 is also small compared to the diagonal elements, i.e., if:

$$E\ [DN_{o_{k-i}} \cdot DN_{o_{k-i}}] >> E\ [DN_{o_{k-i}} \cdot DN_{k-j}] \qquad (55)$$

then equations 51, 52, 53, 54, and 55 imply that the autocorrelation matrix R can be approximated by a scalar matrix.

Therefore, the conclusion that the relative frequency response is determined by the intended narrow-band signal remains valid.

The key assumption above was inequality 55. It shows that the delayed random wide-band interference N_o must significantly decorrelate between the ith and jth taps of the adaptive filter for R to be approximated by a scalar matrix. Since i and j can take on any values, except i = j, the delayed random wide-band interference must significantly decorrelate over one intertap delay time in order for R to look like a scalar matrix. This will insure that the relative frequency response is determined by the intended signal and hence that the ALE will put a "bandpass" around the intended signal.

It is important to note that for both case 1 (S = weak random wide-band intended signal, N_o = strong narrow-band interferer) and case 2 (S = weak narrow-band intended signal, N_o = strong random wide-band interferer) it is the narrow-band signal that the adaptive filter puts a "passband" around. Intuitively this makes sense. A narrow-band deterministic signal can be subtracted from the sum of the same narrow-band deterministic signal and a wide-band random signal. The narrow-band signals can cancel out. Subtracting a wide-band random signal from that same sum will not cancel out the wide-band random signal. Randomness will prevent cancellation.

MEAN SQUARE ERROR AS A PERFORMANCE MEASURE FOR ADAPTIVE ALGORITHMS

Before adaptive algorithms can be investigated, a performance measure or performance function for the adaptive filter must be defined. A very useful and well understood performance function evaluated in this paper is Mean Square Error.

The generation of an adaptive filter error signal is illustrated in Figure 6. The sampled output of the adaptive filter Y_k is subtracted from a sampled desired signal response to generate an error signal. The "desired" response will not usually be the intended signal that is being sought to detect. If the intended signal was known there would be no need for an adaptive filter to detect it. The "desired" response must be related to the intended signal in some manner. For the case of an adaptive noise canceler (illustrated in Figure 2) the "desired" response is the primary input, i.e., the intended signal S plus the interference N_O.

By taking the square of the adaptive filter error function ε_k, ε_k^2 will never be negative and will therefore possess a minimum value.

The adaptive filter should be able to work with random input signals and random "desired" responses as well as with deterministic signals because communications signals are often modeled as random signals. This suggests that an appropriate performance function for an adaptive filter would be the average or mean of the squared error (denoted by $E[\varepsilon_k^2]$). Mean square error can also be interpreted as the average power of the error signal in Figure 5.

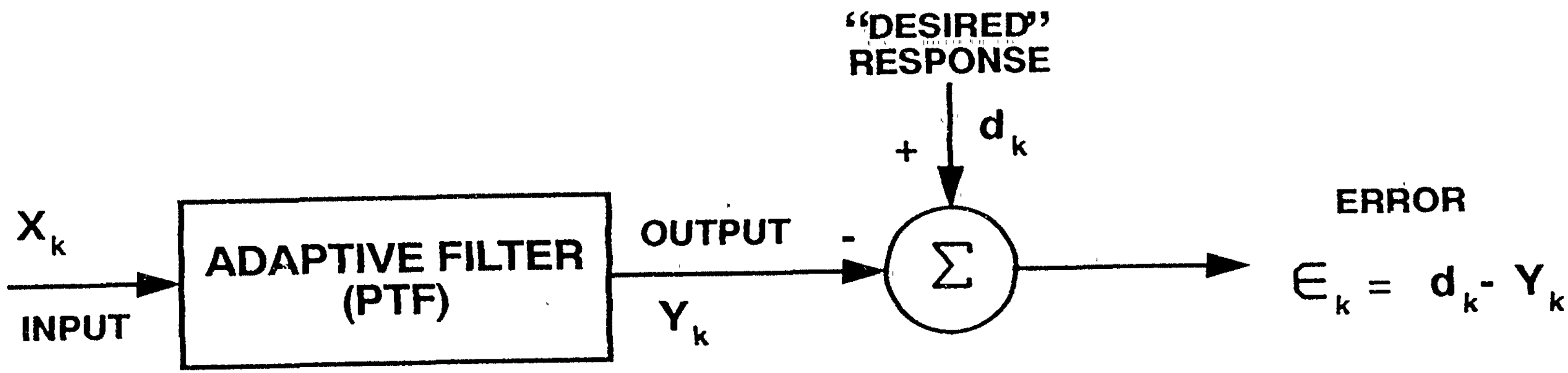


Figure 6. Generation of an Adaptive Filter Error Signal

The mean square error as a function of input signal, "desired" response, and tap weights can be derived using the following definitions. The error signal ε_k at time index k is defined as:

$$\varepsilon_k = d_k - Y_k \tag{56}$$

The output of the PTF is given by:

$$Y_k = W_o X_k + W_1 X_{k-1} + W_2 X_{k-2} + \ldots + W_n X_{k-n} \tag{57}$$

If the column vectors $\vec{W}$ and $\vec{X}_k$ are defined by

$$\vec{W} = \begin{bmatrix} W_o \\ W_1 \\ \cdot \\ \cdot \\ \cdot \\ W_n \end{bmatrix} \quad \text{and} \quad \vec{X}_k = \begin{bmatrix} X_k \\ X_{k-1} \\ \cdot \\ \cdot \\ \cdot \\ X_{k-n} \end{bmatrix}$$

then equation 57 can be expressed as the vector dot product of $\vec{W}$ and $\vec{X}_k$:

$$Y_k = \vec{W}^T \cdot \vec{X}_k \tag{58}$$

where $\vec{W}^T$ is the transpose of $\vec{W}$, i.e., $\vec{W}^T$ is a row vector.

Equation 58 can also be expressed as:

$$Y_k = \vec{X}_k^T \cdot \vec{W} \tag{59}$$

Substituting equations 58 and 59 into equation 56 gives:

$$\varepsilon_k = d_k - \vec{X}_k^T \cdot \vec{W} = d_k - \vec{W}^T \cdot \vec{X}_k \tag{60}$$

Now square equation 60 to get:

$$\varepsilon_k^2 = (d_k - \vec{X}_k{}^T \cdot \vec{W})\ (d_k - \vec{W}^T \cdot \vec{X}_k) \tag{61}$$

$$\varepsilon_k^2 = d_k^2 - d_k \vec{W}^T \cdot \vec{X}_k - d_k \vec{X}_k{}^T \cdot \vec{W} + (\vec{X}_k{}^T \cdot \vec{W})\ (\vec{W}^T \cdot \vec{X}_k)$$

$$\varepsilon_k^2 = d_k^2 + (\vec{W}^T \cdot \vec{X}_k)\ (\vec{X}_k{}^T \cdot \vec{W}) - 2\ d_k\ (\vec{X}^T{}_k \cdot \vec{W}) \tag{62}$$

The second term of equation 62 can be written as

$$(\vec{W}^T \cdot \vec{X}_k) \cdot (\vec{X}_k{}^T \cdot \vec{W}) = \vec{W}^T \cdot [\vec{X}_k\ \vec{X}_k{}^T]\ \vec{W} \tag{63}$$

where $[\vec{X}_k\ \vec{X}_k{}^T]$ is a matrix given by:

$$[\vec{X}_k\ \vec{X}_k{}^T] = \begin{bmatrix} X_k^2, & X_k X_{k-1}, & X_k X_{k-2}, \cdots, & X_k X_{k-n} \\ X_{k-1} X_k, & X_{k-1}^2, & X_{k-1} X_{k-2}, \cdots, & X_{k-1} X_{k-n} \\ \vdots & & & \\ X_{k-n} X_k, & X_{k-n} X_{k-1}, & X_{k-n} X_{k-2}, \cdots, & X_{k-n}^2 \end{bmatrix} \tag{64}$$

Substituting equation 63 into equation 62 gives:

$$\varepsilon_k^2 = d_k^2 + \vec{W}^T \cdot [\vec{X}_k\ \vec{X}_k{}^T]\ \vec{W} - 2\ d_k (\vec{X}_k{}^T \cdot \vec{W}) \tag{65}$$

If it is assumed that ε_k, d_k and $\vec{X}_k$ are statistically stationary (i.e., statistical characteristics are independent of time) and $\vec{W}$ is held constant, then taking the expected value of equation 62 over the time index k yields the following expression for mean square error (MSE):

$$MSE = E[\varepsilon_k^2] = E[d_k^2] + \vec{W}^T \cdot E[\vec{X}_k \vec{X}_k^T]\vec{W}$$

$$- 2E[d_k \vec{X}_k^T] \cdot \vec{W} \qquad (66)$$

where E denotes the expected or mean or average value of the quantity in brackets. In equation 66; $E[\vec{X}_k \vec{X}_k^T]$ is just the input autocorrelation matrix R (see equation 9) and $E[d_k \vec{X}_k^T]$ is just the cross-correlation vector $\vec{P}$ (see equation 10). Equation 66 then becomes:

$$MSE = E[\varepsilon_k^2] = E[d_k^2] + \vec{W}^T \cdot R\vec{W} - 2\vec{P}^T \cdot \vec{W} \qquad (67)$$

It is obvious from equation 67 or 66 that MSE is a quadratic function of the components of the weight vector $\vec{W}$, i.e., the components of $\vec{W}$ appear in equation 67 or 66 raised either to the first or second power. This implies that when MSE is plotted against all the tap weights the result is a hyper paraboloid. If there are n taps in the PTF then a plot of MSE versus tap weights yields an (n + 1) dimensional "parabola." This plot is known as a performance surface.

An n + 1 dimensional parabola can be thought of as an (n + 1) dimensional "bowl". This "bowl" must be concave upward; otherwise there would be weight settings that would result in a negative MSE (i.e., negative average error signal power). This is impossible with real physical signals. Since the MSE is a quadratic function, this implies that there is a single point at the bottom of the MSE performance surface "bowl." This point is the minimum MSE. The objective of all adaptive algorithms is to drive the weights and the resulting MSE toward this point.

Equation 8 for the optimal weight vector W* provides a direct method of locating the bottom of the MSE performance surface bowl. When we assume a weight vector W = W* then the mean square error is at its minimum. This is known as the direct or matrix inversion algorithm. This algorithm has several severe drawbacks associated with it:

1. If the PTF has n taps, then (n+1) (n+4) / 2 autocorrelation and cross-correlation measurements must be made in order to determine R and $\vec{P}$. Such measurements must be repeated whenever the input signal statistics change with time.

2. The autocorrelation matrix must then be inverted.

3. "Implementing a direct solution requires setting weight values with a high degree of accuracy in open loop fashion, whereas a feedback approach provides self correction of inaccurate settings thereby giving tolerance to hardware error."[5] In other words, because equation 8 has no feedback from the error output, highly accurate weight values are required.

When the number of weights is large or the input data rate (or hopping rate for frequency hopping radios) is high, then 1 and 2 above imply severe computational and time requirements on any direct solution. The processor implementing a matrix inversion algorithm might not be able to implement it fast enough for the algorithm to be of any use. Because of these problems, no adaptive algorithms that require the measurement of an autocorrelation matrix or the computation of its inverse were investigated.

Two types of adaptive algorithms that do not require any knowledge of the autocorrelation matrix are the methods of Steepest Descent and Random Search.

METHOD OF STEEPEST DESCENT

Before introducing the method of steepest descent for an arbitrary number of tap weights (or equivalently an arbitrary number of dimensions in the mean square error performance surface) it is helpful to consider the method of steepest descent for the simplest case: just one weight.

The one weight (univariable) performance surface, which is a parabola, is shown in Figure 7.

The method of steepest descent does not require knowledge of the autocorrelation matrix R or the cross-correlation vector $\vec{P}$. Since R and $\vec{P}$ are unknown, equation 67 cannot be used to define the MSE performance surface. But since mean square error can also be interpreted as the average power of the error signal, MSE can be measured.

In order to find W*, the weight that causes the MSE to be minimized, an arbitrary weight value W_o is initially assumed. The average power of the error signal is then measured in order to determine the MSE at W_o, i.e., one point on the MSE performance "surface" shown in Figure 7 has been located. The ability to locate points on the MSE performance "surface" allows measurement of the slope of the parabola at W_o (the method by which the slope is measured depends on the type of steepest descent algorithm used).

A new weight value W_1 is then chosen equal to the initial value W_o plus an increment proportional to the negative of the

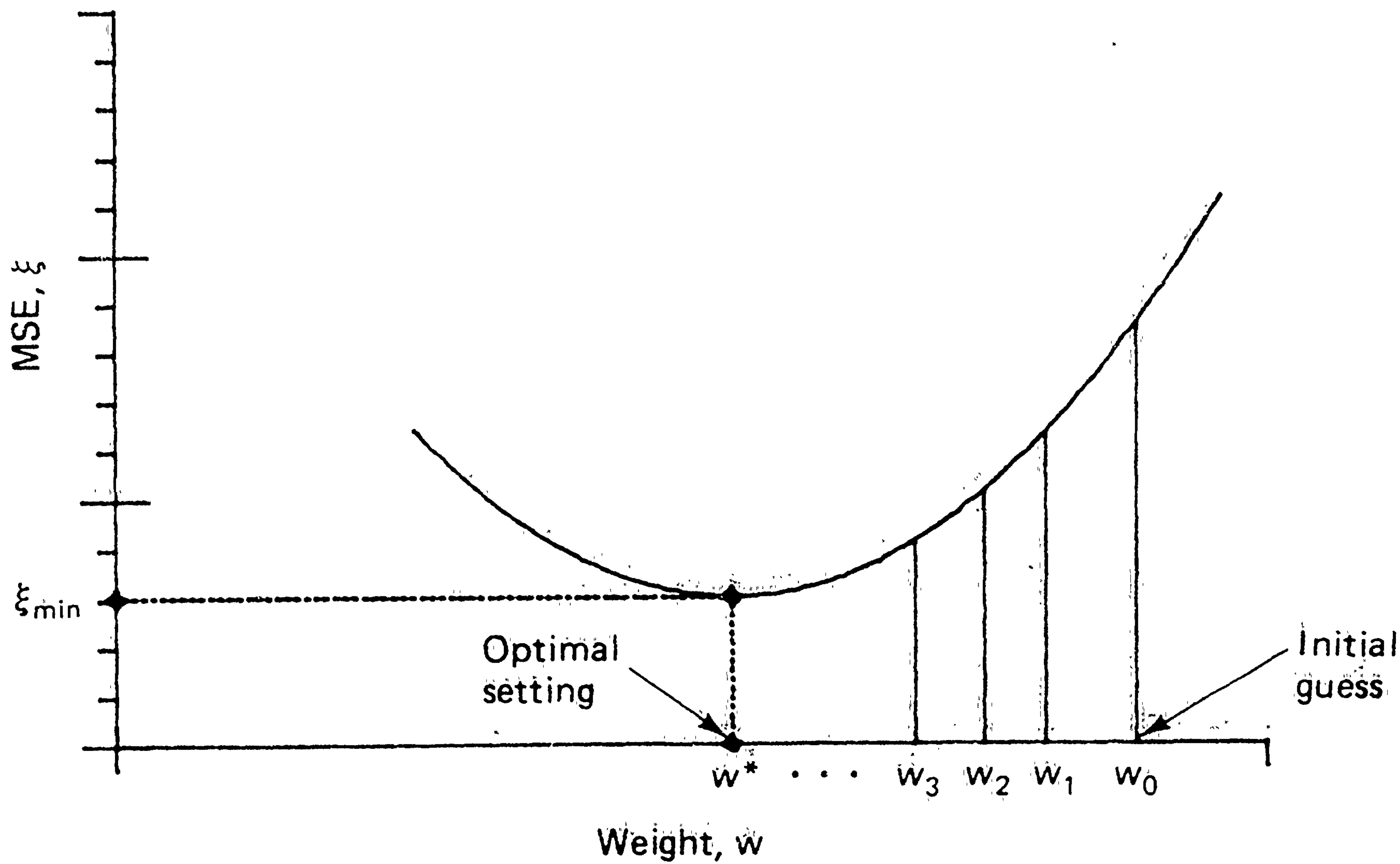


Figure 7. Gradient Search of Unvariable Performance Surface (From Ref. 1, Page 47)

slope at W_o

$$W_1 = W_o + \mu \text{ (-slope)} \tag{68}$$

The point on the performance surface corresponding to W_1 is lower down on the parabola than the point corresponding to W_o. It is closer to the minimum than the first point. Another new value, W_2, is then derived in the same way by measuring the slope of the parabola at W_1, i.e.,

$$W_2 = W_1 + \mu \text{ (-slope)} \tag{69}$$

This procedure is repeated until the slope of the parabola at the iterated point is zero. It is obvious from Figure 7 that when the slope of the parabola is zero, then W^*, the weight that causes the MSE to be minimized, has been identified. To summarize, for a one weight filter with a parabolic error surface, the negative of the slope of the parabola is used to "slide" down to the bottom of the "bowl."

For a filter with n taps and an n + 1 dimensional hyper paraboidal mean square error surface, the objective is still to "slide" down the error surface to the bottom of the "bowl."

In order to identify (at any given point on the MSE surface) the direction in which to slide, the negative gradient vector of the MSE surface is used. The gradient of the MSE surface at a given point on the surface gives the direction in which the MSE is increasing fastest at that point. The negative of the gradient is the direction in which the MSE is decreasing fastest. It points the way to the steepest (and "fastest") descent down the MSE "bowl." Hence the name "Method of Steepest Descent."

The gradient ∇ of the MSE surface is defined as the vector

$$\nabla = \left[\frac{\partial(MSE)}{\partial W_o}, \frac{\partial(MSE)}{\partial W_1}, \ldots., \frac{\partial(MSE)}{\partial W_n} \right] \tag{70}$$

i.e., each component of ∇ is a partial derivative of the MSE with respect to a given weight.

The method of steepest descent can be expressed by the following algorithm:

$$\vec{W}_{k+1} = \vec{W}_k + \mu\,(-\nabla_k) \tag{71}$$

where

$\vec{W}_k$ = the weight vector at the k<u>th</u> iteration, i.e., the set of tap weights used on the k<u>th</u> iteration.

$\vec{W}_{k+1}$ = the weight vector at the k+1<u>th</u> iteration

∇_k = the gradient at the k<u>th</u> iteration point on the MSE performance "surface"

μ = a constant that regulates the step or increment size of the weight vector change. It determines how far to "slide" down the performance surface before another iteration is performed.

Equation 71 is a direct generalization of the one dimensional case (equations 68 and 69). For any given set of tap weights, $\vec{W}_k$, a new set $\vec{W}_{k+1}$ can be computed (via equation 71) that yields a smaller mean square error. In order to use equation 71 it must be possible to compute the gradient ∇_k at the k<u>th</u> iteration point. The manner in which the gradient is computed depends on the specific steepest descent algorithm that is used. All steepest descent algorithms, however, use the fact that mean square error can be interpreted as the average power of the error signal to locate points on the MSE performance surface and to ultimately use these points to compute the gradient. To summarize equation 71, the defining equation for the method of steepest descent, allows an iterative approach to the optimal weight vector $\vec{W}^*$ without any knowledge of the autocorrelation matrix R or the cross-correlation vector P. The only prerequisite for using equation 71 is the ability to measure average error signal power.

GRADIENT ESTIMATION

The two most widely used methods for estimating the gradient at a given point on the mean square error surface are: the Differential Steepest Descent (DSD) algorithm and Widrow's Least Mean Square (LMS) algorithm.

DIFFERENTIAL STEEPEST DESCENT ALGORITHM

In the DSD algorithm, each of the partial derivatives in equation 70 are estimated by the method of symmetric differences illustrated in Figure 8. To calculate $\partial(MSE)/\partial W_i$ at a given value of $W_i = W_{Given}$, all the weights except W_i are held constant. As per Figure 8, the mean square error is "measured" at $W_i = W_{Given} + \delta$ and at $W_i = W_{Given} - \delta$. The slope of the line between the two points is then calculated via equation 72

$$\text{slope} = \frac{MSE(W_{Given} + \delta) - MSE\ (W_{Given} - \delta)}{2\ \delta} \tag{72}$$

This slope is an approximation of $\partial(MSE) / \partial W_i$ at $W_i = W_{Given}$.

The MSE terms in equation 72 above are just estimates of the true MSE based on measurement of the average error signal power. There will be an error associated with each MSE measurement. This means that $\partial(MSE) / \partial W_i$ given by the slope in equation 72 will have an error associated with it. Since δ is small, MSE $(W_{Given} + \delta)$ and MSE $(W_{Given} - \delta)$ will be very close to each other. When the two MSE values are subtracted, as in equation 72, the resulting error (on a percentage basis) becomes greatly magnified. The only way to reduce this subtraction or slope error is to reduce the MSE error. This is done by repeated MSE measurement at both $W_{Given} + \delta$ and at $W_{Given} - \delta$. In other words, the error signal average power must be measured M times at both $W_{Given} + \delta$ and at $W_{Given} - \delta$; M will be determined by the accuracy requirements of the particular application. Therefore, DSD algorithm requires 2M error signal average power measurements per tap per iteration.

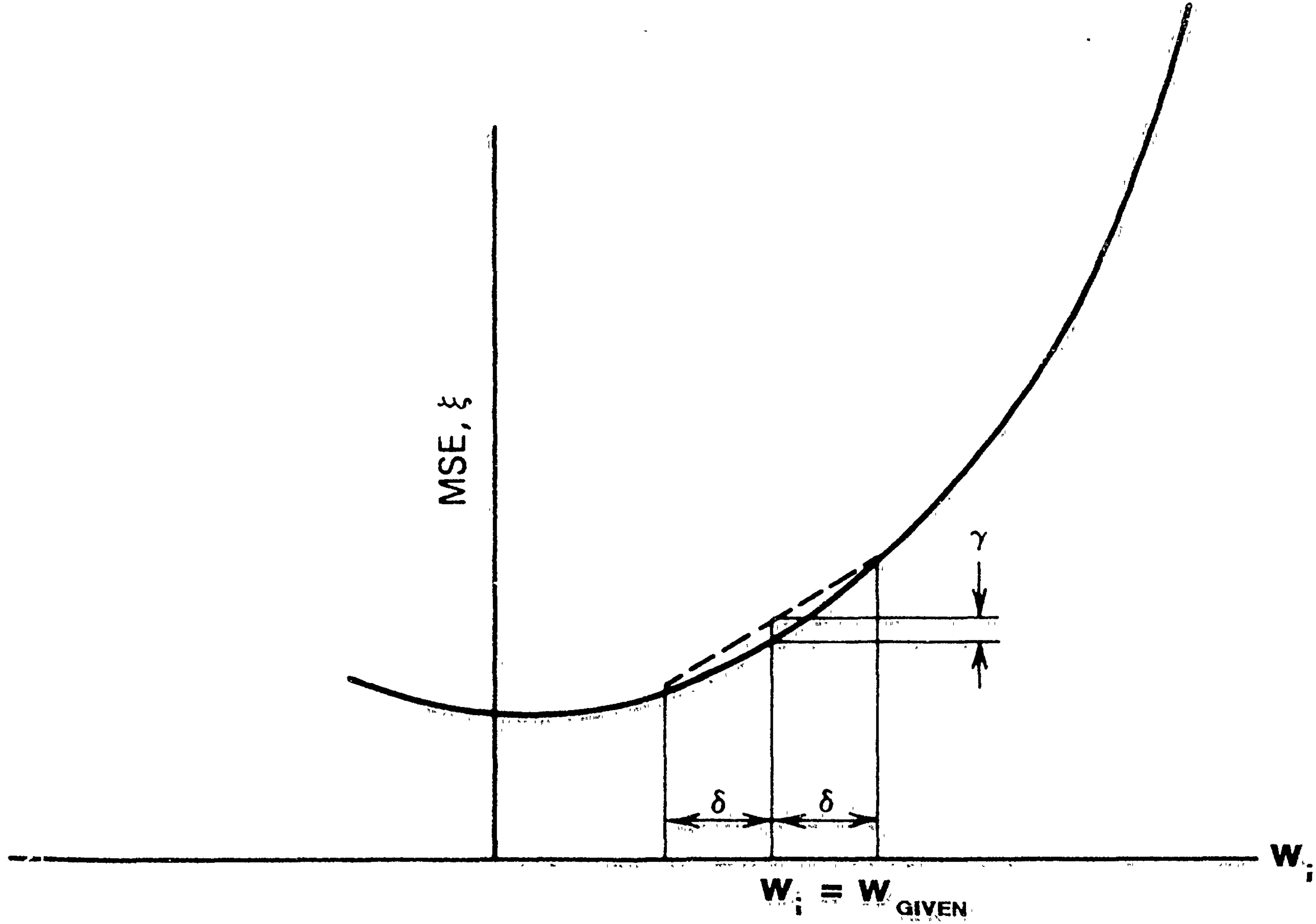


Figure 8. Gradient Estimation by Way of Direct Measurement (From Ref. 4, Page 179)

In the DSD algorithm, once the gradient has been approximated (via equation 70) by the method of symmetric differences it is substituted into the defining equation (equation 71) for the method of steepest descent and a new set of tap weights are calculated.

LEAST MEAN SQUARE (LMS) ALGORITHM

In the LMS or Widrow's algorithm it is assumed that the adaptive filter is an adaptive linear combiner (see Figure 9). If data are acquired and input in parallel to an adaptive linear combiner, the structure in Figure 9a is used. For serial data input the structure in 9b is used. Note that Figure 9b is just a tapped delay line or transversal filter. It is further assumed that a "desired" response signal is available. These two assumptions were not made for the DSD algorithms. So DSD is more general than LMS, i.e., it is not tied to a single filter structure. LMS is only applicable to the adaptive linear combiner.

In the LMS algorithm, each of the partial derivatives in equation 70 can be estimated by assuming that the mean square error (MSE) can be estimated by a single measurement of the error, i.e.,

$$\text{MSE} \approx \varepsilon_k^2 \tag{73}$$

where ε_k = single measurement of the error at the k<u>th</u> iteration. Equation 73 is the key assumption in the LMS algorithm. Substituting equation 73 into equation 70 results in:

$$\nabla_k = \left[\frac{\partial(\varepsilon_k^2)}{\partial W_o}, \frac{\partial(\varepsilon_k^2)}{\partial W_1}, \ldots, \frac{\partial(\varepsilon_k^2)}{\partial W_n} \right] \tag{74}$$

where ∇_k is the gradient of the MSE performance surface at the kth iteration point.

$$\frac{\partial \varepsilon_k^2}{\partial W_i} = \frac{d\varepsilon_k^2}{d\varepsilon_k} \cdot \frac{d\varepsilon_k}{dW_i} = 2\varepsilon_k \frac{d\varepsilon_k}{dW_i} \tag{75}$$

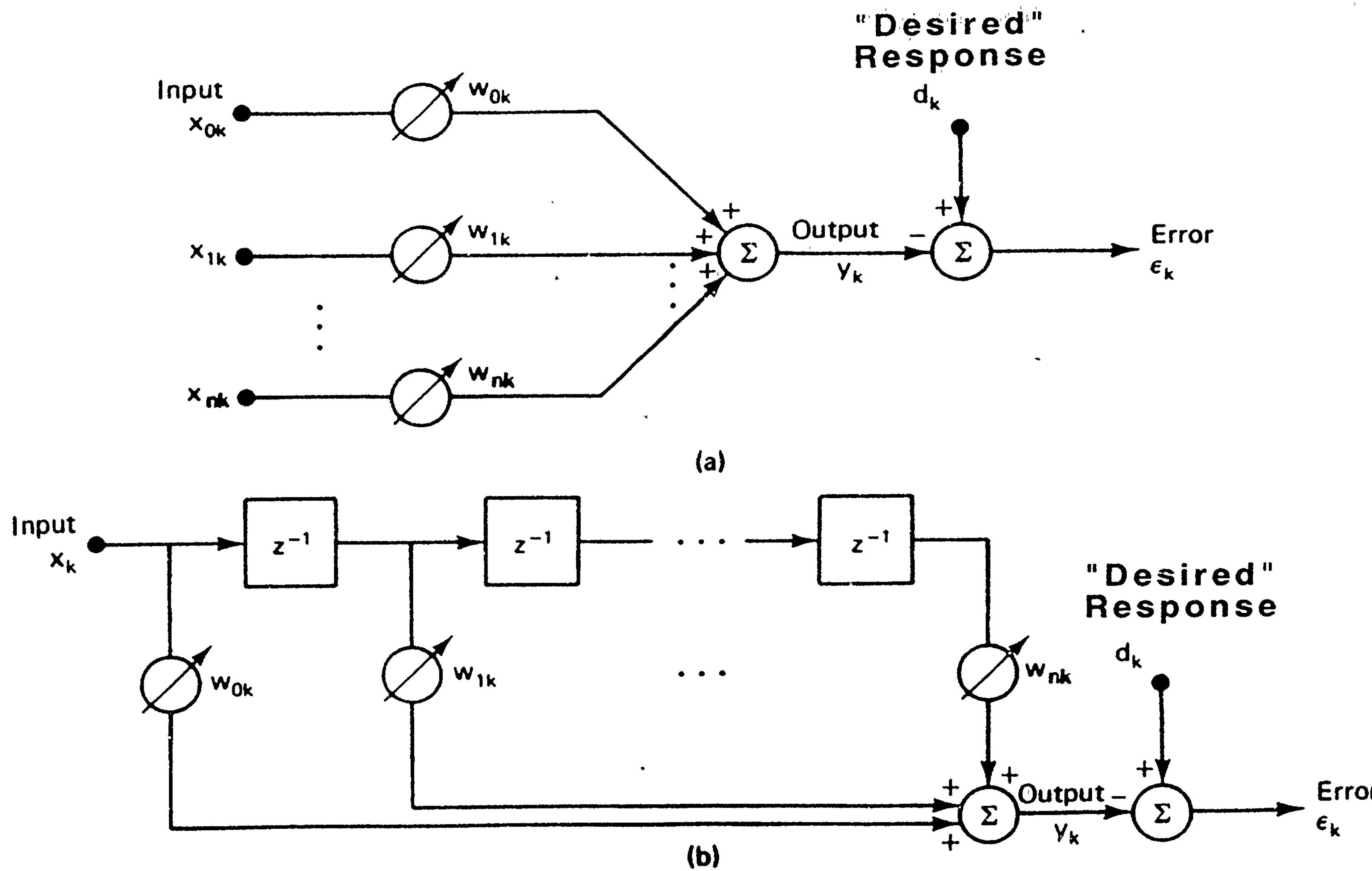


Figure 9. The Adaptive Linear Combiner: (a) In General Form; (b) As a Transversal Filter (From Ref. 1, Page 101)

Since an adaptive linear combiner filter structure was assumed, this implies that:

$$\varepsilon_k = d_k - \sum_{i=o}^{N} X_{ki}\, W_{ki} \tag{76}$$

where X_{ki} = signal at tap i during the k<u>th</u> iteration

W_{ki} = tap weight at tap i during the k<u>th</u> iteration.

Taking the derivative of equation 76 implies

$$\frac{d\varepsilon_k}{dw_{ki}} = -X_{ki} \tag{77}$$

In equation 77, in order to be consistent with equation 75, we will change X_{ki} to X_i and W_{ki} to W_i. Equation 77 then becomes:

$$\frac{d\varepsilon_k}{dW_i} = -X_i \tag{78}$$

Substituting equation 78 into equation 75 gives:

$$\frac{\partial\varepsilon_k^2}{\partial W_i} = -2\varepsilon_k X_i \tag{79}$$

Substituting equation 79 into equation 74 gives:

$$\nabla_k = [-2\varepsilon_k X_o,\ -2\varepsilon_k X_1,\ \ldots,\ -2\varepsilon_k X_n] = 2\varepsilon_k \vec{X}_k \tag{80}$$

where $\vec{X}_k = [X_o, X_1, \ldots, X_n]$, i.e., $\vec{X}_k$ is a vector representing the tap values at the k<u>th</u> iteration.

The method of steepest descent is defined by equation 71:

$$\vec{W}_{k+1} = \vec{W}_k + \mu \, (-\nabla_k) \tag{71}$$

Substituting equation 80 into equation 71 gives:

$$\vec{W}_{k+1} = \vec{W}_k + 2\mu \, \varepsilon_k \vec{X}_k \tag{81}$$

Equation 81 is the LMS algorithm.

The LMS algorithm is very easy to compute, and, given the right hardware, it can be done very quickly. It does not require off-line gradient estimation or repetitive error measurements as in the DSD algorithm. In addition, for a given iteration, all of the signal values (X_0, X_1, ..., X_n) at the individual taps can in theory be measured in parallel at the same time. This allows a parallel measurement of the gradient (via equation 80). This is in contrast to the DSD algorithm where each partial derivative ($\partial(MSE) / \partial W_i$) must be measured sequentially in order to compute the gradient via equation 70. Thus the LMS algorithm is potentially much faster than the DSD algorithm.

RANDOM SEARCH ALGORITHM

So far, two adaptive algorithms have been considered: Least Mean Square (LMS) and Differential Steepest Descent (DSD). LMS adapts faster than DSD. LMS does, however, require knowledge of the signal value at each tap of the programmable transversal filter (PTF). This requirement adds additional complexity to the adaptive filter. An auxiliary PTF has to be added to the adaptive filter. The tap signal values are measured on the auxiliary PTF so as not to interfere with the operation of the "main" PTF. DSD is more general than LMS, but it requires that all the partial derivatives of mean square error with respect to the weights ($\partial(MSE) / \partial W_i$) be measured (sequentially). In addition, the MSE must be measured a number of times to insure accuracy. Random search algorithms do not require knowledge of the signal at each tap of the PTF as does LMS. Nor do they require measurement of $\partial(MSE) / \partial W_i$ as does DSD. Random search algorithms tend to be slower than LMS, but faster than DSD. DSD, however, will outperform random search algorithms in terms of certain performance measures that are beyond the scope of this report. Random search algorithms are useful when LMS cannot be applied, i.e., when the adaptive filter is not an adaptive linear combiner or PTF or when its complexity is not "affordable".

One of the most efficient random search algorithms is the Linear Random Search (LRS) algorithm. In LRS: "a small random change U_k is tentatively added to the weight vector at the beginning of each iteration. The corresponding change in mean square error performance is observed. A permanent weight vector change, proportional to the product of the change in performance and the initial tentative change, is then made."[7]

The new weight vector generated by the LRS algorithm is given by

$$W_{k+1} = W_k + \mu\,[\hat{\xi}\,(W_k) - \hat{\xi}\,(W_k + \mu_k)\,]\,\mu_k \tag{82}$$

where:

μ_k is a random vector.

$\hat{\xi}\,(W_k)$ is an estimate of mean square error at $W = W_k$ based on N samples.

$\hat{\xi}\,(W_k + \mu_k)$ is an estimate of mean square error at $W = W_k + \mu_k$ based on N samples.

μ is a design constant affecting stability and rate of adaptation.

PTF HARDWARE IMPLEMENTATION

Although the primary purpose of this report is to describe the theoretical principles of adaptive noise canceling, this section will be devoted to a description of a SAW device implementation of a PTF.

"Several programmable SAW filters have been reported in the literature.[10-13] Most are used for match filter operation. A SAW/FET approach demonstrated 50 MHz of bandwidth centered at 150 MHz. However, tap control range was limited to 16 dB and single tap insertion loss was 80 dB.[14] A monolithic GaAs approach in which the SAW and the FETs are implemented on the same substrate has demonstrated 58 dB dynamic range at 500 MHz over a 50 MHz bandwidth.[15,16,17]"

A promising approach suitable for use in an adaptive noise canceler, is a hybrid programmable transversal filter (HPTF).[16,17] All programmable transversal filter designs reported to date are severely limited by poor tap weight control range (which limits filter sidelobe performance) and poor dynamic range (which limits sensitivity). The HPTF solves both of these problems by combining a $LiNbO_3$ SAW device for high dynamic range with GaAs dual-gate FETs for high tap weight control range. Measured tap weight control range (70 dB) and dynamic range (85 dB over a 100 MHz bandwidth) are high enough to meet many system requirements.

"The HPTF consists of a tapped SAW delay line whose output electrodes are connected to an array of tap weight control dual-gate FETs (Figure 10). The signal is applied to an input transducer, which generates a surface acoustic wave that propagates down the substrate. An array of output transducers transforms this acoustic wave back into electrical signals that are delayed copies of the original input. Each output transducer is connected to the input (gate-1) of a dual-gate FET (DGFET) tap weight control amplifier. The tap weight is controlled by gate-2 voltage. The DGFET outputs (drains) are connected to a common current summing bus. The transversal filter can now be identified by the process of shift, multiply and sum. Negative tap weights are generated with a second DGFET array whose output is inverted by an external differential amplifier. This alleviates the need for an invertor at each tap."[16,17]

The maximum power handling capability of an HPTF is limited by the power that can be safely applied to the SAW input transducer (about +20 dBm).

Typically, when used either as a bandpass or notch filter, an HPTF can reduce interfering signals by 40-50 dB. A single tap weight on the HPTF can be changed in approximately 1 microsecond. To change an entire set of tap weights to a second set will usually take much longer. A 16-tap HPTF has 16 weights to be changed. If this is done serially, then the single tap switching time of 1 microsecond must be multiplied by 16. In reality, a 128 tap filter will be needed. So a 1 microsecond switching time per tap must be multiplied by 128. In addition, a controller must address and transfer the tap weights to the HPTF. The transfer time per tap could be much larger than the single tap switching time. If the HPTF is included in an adaptive noise canceler, then a number

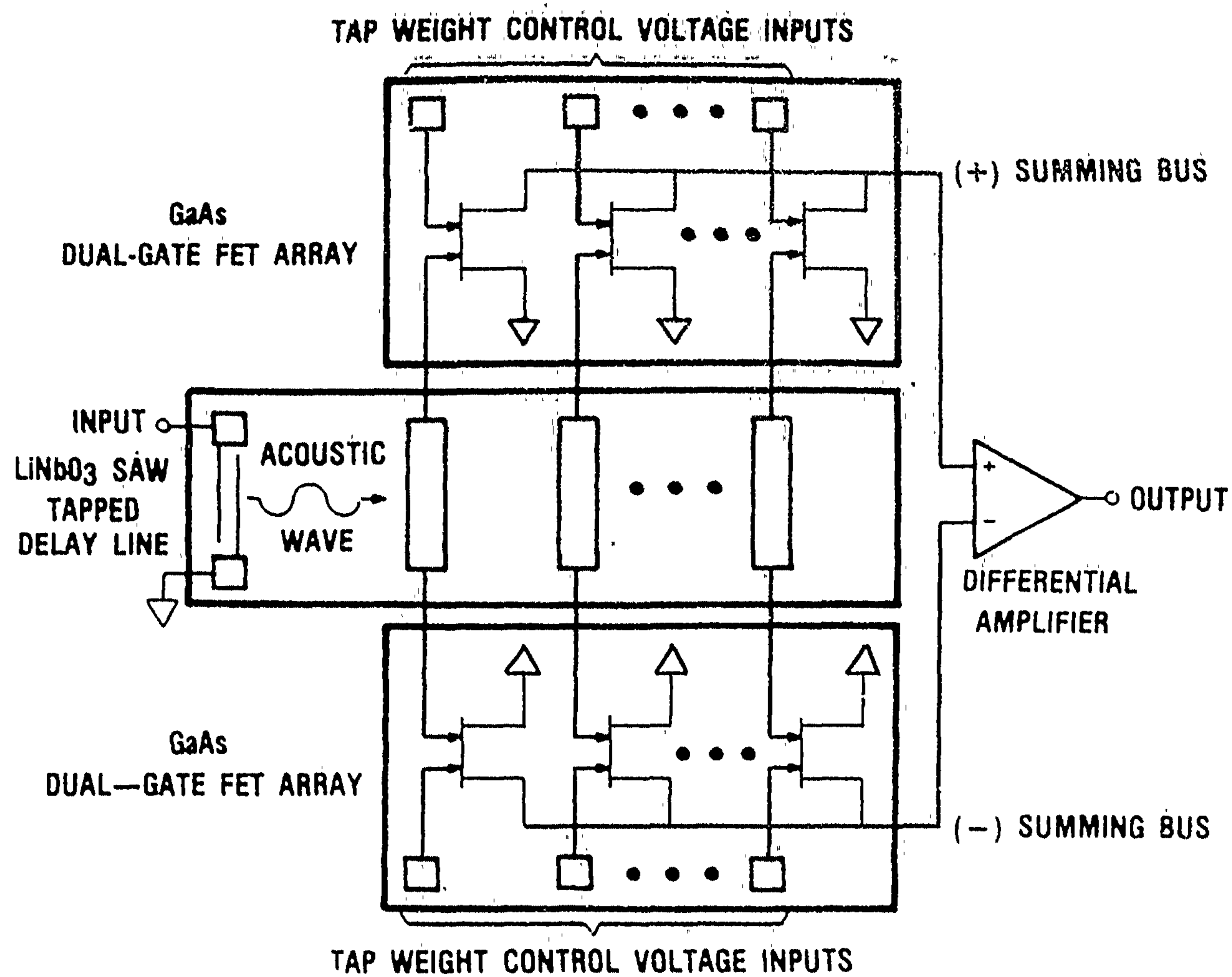


Figure 10. Hybrid Programmable Transversal Filter (HPTF) Concept (From Ref. 16 and 17)

of tap weight sets will have to be transferred from the controller to the HPTF. The output power of the HPTF will have to be measured and transferred to the controller.

If Widrow's algorithm is used, the signals on each tap have to be measured and transferred to the controller. For each tap, the controller will then have to calculate a new weight. The speed of the calculation will depend on the speed of the controller. All this overhead implies a much longer time to achieve adaptive convergence (in an adaptive noise canceler) than to simply switch a single tap weight.

It is expected that a 128-tap HPTF type filter will be able to achieve 30 dB of filtering (in an adaptive noise canceler configuration) in approximately 1 millisecond. A 128 tap HPTF type filter is currently being developed for ETDL by Texas Instruments under Contract No. DAAL01-88-C-0831.

CONCLUSIONS

The theoretical principles developed within this report (i.e., the mathematical structure of the autocorrelation matrix R, the cross-correlation vector P, and the Wiener or optimal weight vector W*) imply that adaptive noise canceling is a viable method of separating weak and strong signals.

If both the intended and interfering signals are narrow-band, then an adaptive noise canceler with a single input is the appropriate filter structure. This is because, as shown in the "Analysis of an Adaptive Noise Canceler with a Single Input" section, the optimal weight vector W* will be dominated or determined by the strong interferer. This will cause the programmable transversal filter (PTF) to form a bandpass around the strong interferer, pass the interferer, and reject the intended signal. The output of the PTF (the filtered interfering signal) is then subtracted from the signal plus interference at the output power combiner and yields the intended signal.

For separating narrow-band and random wide-band signals, the adaptive noise canceler must be configured as an adaptive line enhancer. As was shown in the "Analysis of an Adaptive Line Enhancer" section, an appropriate delay before the PTF in the ALE will cause a passband to appear (in the PTF frequency response curve) around the narrow-band signal. Most of the random wide-band signal will then be filtered out. The resulting narrow-band signal will be subtracted from the sum of both signals (at the output power combiner). The output of the combiner is the wide-band signal. In this way signal separation is achieved.

The choice of an adaptive algorithm for an adaptive noise canceler depends on several factors. If adaptation time is most important, then Least Mean Squares (LMS) should be chosen. If simplicity and hardware costs are the driving factors, then a random search algorithm such as the Linear Random Search (LRS) should be chosen. If the adaptive filter is not an adaptive linear combiner or programmable transversal filter, then the Differential Steepest Descent (DSD) algorithm or a Random Search Algorithm would be appropriate choices since neither of these algorithms assume a transversal filter structure for the adaptive filter in the ALE (LMS algorithm does assume that the adaptive filter is a transversal filter).

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